Determination of Backbending in $^{122-130}$Ba Even-Even Isotopes

I. M. Ahmed                             W. M. Najeeb
Department of Physics                   Department of Physics
College of Education                        College of Science
University of Mosul                      University of Mosul

Received 21 / 05 / 2007  Accepted 15 / 08 / 2007

Abstract

The $\gamma$-unstable O(6) limit of the interacting boson model IBM-1 has been applied successfully to determine the backbending in weakly deformed $^{122-130}$ Ba even-even isotope. The application of this limit has showed successes in determining the backbending in the energy levels of the isotopes under consideration through the good coincidence with the experimental results.
**Introduction:**

The backbending phenomenon occurs due to the rapid increase of the moment of inertia with rotational frequency toward the rigid value [1]. When the rotational energy exceeds the energy needed to break a pair of nucleon, the unpaired nucleon goes into a different orbit causing a change in the moment of inertia [2], other proposed explanation such as rotational alignment [3], and centrifugal stretching, [4] along with the former, could be described in terms of band crossing [5].

The main purpose of the present work is to investigate the backbending effect in $^{122-130}$Ba even-even isotopes, using $\gamma$-unstable $O(6)$ limit of the interacting boson model IBM-1 [6].

Several studies have been performed to investigate the backbending effect in some even-even Ba isotopes [7 and 8].

**Theory:**

**Yrast levels and backbending**

The lower energy level for each spin is called yrast level [9]. One of the interesting observations made on yrast bands is the presence of small and sudden changes in the moment of inertia on a plot of $E_J$ as a function of $J(J+1)$. The sudden change are usually too insignificant to be noticeable. However if the moment of inertia is plotted against the square of the frequency of the rotation, a local variation in the moment of inertia around a significant high spin would be occurred.

The rotational energies are given by [2]:

$$E_J = \frac{\hbar^2}{2\mathfrak{I}} J(J+1)$$

(1)

$\mathfrak{I}$ is the moment of inertia and $J$ is the spin of the state.

The energy of a transition from state $J$ to the next lower state $J-2$ is given by [9]:

$$E_J - E_{J-2} = \frac{\hbar^2}{2\mathfrak{I}} (4J - 2)$$

(2)

and the local value of the moment of inertia will be:

$$\frac{2\mathfrak{I}}{\hbar^2} = \frac{4J - 2}{E_J - E_{J-2}}$$

(3)
The rotational frequency $\omega$ is not a quantity that can be measured but may be inferred by analogy with classical rotational frequency through the relation [9]:

$$\hbar \omega = \frac{dE}{d\sqrt{J(J+1)}}$$  

------------------------(4)

For a $K=0$ band, the usual case for the yrast band in even-even nuclei; the value may be approximated by the difference between $E_J$ and $E_{J-2}$ to:

$$\hbar \omega \approx \left| \frac{\Delta E}{\Delta \sqrt{J(J+1)}} \right|^{\frac{1}{2}} \frac{1}{J \rightarrow \infty} \frac{1}{2} (E_J - E_{J-2})$$  

------------------------(5)

$\gamma$-unstable O (6) limit

A unified description of collective nuclear states was proposed by Arima & Iachello [10] where each nucleon pair is considered as a boson [11], the low-lying collective states can be classified according to the totally symmetric irreducible representation [N] of the group SU (6).

There are a dynamical symmetries results from the degeneracy of the unitary group SU(6) to three limits with three analytical solution [11]. $\gamma$-unstable O(6) limit is the one among these limits ,where the group SU(6) degenerate to the chain [12] :

$$SU(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2)$$  

------------------------(6)

The wave function which describes this limit is given by the quantum numbers $|N\sigma \tau \upsilon \Delta L_M \rangle$ where $\sigma$ is used to characterize the totally symmetric irreducible representations of O (6). This quantum number may take the following values [13]

$$\sigma = N, N-2, \ldots, 0 \ or \ 1 \ for \ N = even \ or \ odd$$  

------------------------(7)

and $\tau$ characterizes the totally symmetric irreducible representation of O (5) where [12]

$$\tau = \sigma, \sigma - 1, -2, \ldots, 0, \ldots, \ldots (8)$$

The quantum number $\upsilon \Delta$ counts d-boson triplets that coupled to zero angular momentum; and finally $L$ and $M_L$ are the total angular momentum and its Z-component.
The values of \( L \) which are contained in each representation \( \tau \) of \( O(5) \) are obtained by partitioning \( \tau \) as [12]
\[
\tau = 3\nu_\Delta + \lambda \quad \nu_\Delta = 0,1 \quad \text{(9)}
\]
and taking
\[
L = 2\lambda, 2\lambda - 2, \ldots, \lambda + 1, \lambda \quad \text{.........(10)}
\]
Note that the absence of \( L = 2\lambda - 1 \) value and the Hamiltonian of this group is given by [13]
\[
H_{O(6)} = aP^+.P^+ + bT_3^+.T_3^+ + cL^+.L^+ \quad \text{.........(11)}
\]
This Hamiltonian has an eigen values [14]
\[
E(\sigma, \nu, L) = K_3[N(N + 4) - \sigma(\sigma + 4) + K_4\nu(\nu + 3) + K_5L(L + 1)] \quad \text{.........(12)}
\]
where \( K_3, K_4 \) and \( K_5 \) are the strength parameters for each term.

**Results and Discussions:**

\( \gamma \)- unstable \( O(6) \) limit of the interacting boson model IBM-1 has been used to determine the energy levels of the yrast band for \( A=122-130 \) even-even Ba isotopes. The first term of eqn. (13) has no significant effect since \( \sigma = N \) for the yrast band. The double solution of eqn. (13) for each isotope gives two values for each parameters \( K_4 \) and \( K_5 \) for both bands, the ground state band and the other band, which gathered to form the yrast levels.

Table (1) shows the values of \( K_4 \) and \( K_5 \) for the ground state band and the other band for each isotope under consideration. So through knowing the values of \( K_4 \) and \( K_5 \), a determinations of the yrast levels can be performed by using eqn.(12) for each isotopes. Table (2) shows the available measured and present calculation of the yrast levels where a good agreement has been found.

Fig.(1) shows the experimental and the calculated energy levels \( E_J \) in (keV) against \( J(J+1) \) for each isotopes and it is obvious that the backbending phenomenon is observed for each isotopes under consideration except \( ^{122}\text{Ba} \).

A calculation of \( \frac{2\gamma}{\hbar^2} \) has been done by using eqn.(3), and \( (h\omega)^2 \) is calculated using eqn.(5). Fig.(2) shows the experimental and the calculated \( \frac{2\gamma}{\hbar^2} \) in keV against \( (h\omega)^2 \) in (keV)\(^2\) for all used isotopes and the backbending phenomenon is clearly observed for all isotopes except \( ^{122}\text{Ba} \).
The present work suggests that the interacting boson model IBM-1, for $\gamma$-unstable O(6) limit is a successful tool to study the yrast levels in the transition nuclei as in $^{122-130}$Ba isotopes. Using this model gives a fairly accurate description of the backbending phenomenon in $^{122}$Ba even-even isotopes except $^{122}$Ba, and this and this ascribes to the ultimate available values of the energy levels of $^{122}$Ba isotope, while the energy levels for the other isotopes ($^{124-130}$Ba) are more available to determine the backbending phenomenon. The appearance of the backbending depending on the amount of deformation of the isotope.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>g.s band</th>
<th>other.bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_4</td>
<td>K_5</td>
<td>K_4</td>
</tr>
<tr>
<td>$^{122}$Ba</td>
<td>28.9</td>
<td>13.4</td>
</tr>
<tr>
<td>$^{124}$Ba</td>
<td>41.85</td>
<td>10.433</td>
</tr>
<tr>
<td>$^{126}$Ba</td>
<td>45.9</td>
<td>12.067</td>
</tr>
<tr>
<td>$^{128}$Ba</td>
<td>61.0</td>
<td>6.667</td>
</tr>
<tr>
<td>$^{130}$Ba</td>
<td>94.45</td>
<td>-3.466</td>
</tr>
</tbody>
</table>

Table (1) the value of K_4 and K_5 parameters in keV for $^{122-130}$Ba even-even isotopes
### Table (2): Comparison between the measured and calculated energy yrast levels in (keV)

<table>
<thead>
<tr>
<th>122 Ba</th>
<th>124 Ba</th>
<th>126 Ba</th>
<th>128 Ba</th>
<th>130 Ba</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eexp</td>
<td>Ecalc</td>
<td>Eexp</td>
<td>Ecalc</td>
</tr>
<tr>
<td>2+</td>
<td>196</td>
<td>196</td>
<td>229.84</td>
<td>229.84</td>
</tr>
<tr>
<td>4+</td>
<td>570</td>
<td>557</td>
<td>651.65</td>
<td>627.0</td>
</tr>
<tr>
<td>6+</td>
<td>1083</td>
<td>1083</td>
<td>1228.38</td>
<td>1191.0</td>
</tr>
<tr>
<td>8+</td>
<td>1704</td>
<td>1703</td>
<td>1923.23</td>
<td>1923.23</td>
</tr>
<tr>
<td>10+</td>
<td>2398</td>
<td>2366</td>
<td>2687.4</td>
<td>2771</td>
</tr>
<tr>
<td>12+</td>
<td>3124</td>
<td>3123</td>
<td>3436.2</td>
<td>3477</td>
</tr>
<tr>
<td>14+</td>
<td>4125.9</td>
<td>4234</td>
<td>4420</td>
<td>4535</td>
</tr>
<tr>
<td>16+</td>
<td>4842.5</td>
<td>5042</td>
<td>5245</td>
<td>5391</td>
</tr>
<tr>
<td>18+</td>
<td>5763.2</td>
<td>5900</td>
<td>6195</td>
<td>6253</td>
</tr>
<tr>
<td>20+</td>
<td>6711.1</td>
<td>6810</td>
<td>7183</td>
<td>7183</td>
</tr>
<tr>
<td>22+</td>
<td>7716.4</td>
<td>7770</td>
<td>8145</td>
<td>8159</td>
</tr>
<tr>
<td>24+</td>
<td>8994.4</td>
<td>8781</td>
<td>9202</td>
<td>9182</td>
</tr>
</tbody>
</table>

- a) Ref (15)
- b) Ref (16)
- c) Ref (17)
- d) Ref (18)
- e) Ref (19)
Fig(1): The energy levels $E(j)$ in (keV) against $j(j+1)$
Fig(2): The $\frac{2\xi}{\hbar^2}$ in (keV)$^{-1}$ against $(\hbar \omega)^2$ in (keV)$^2$.
References:


